NASA TECHNICAL TRANSLATION NASA TT F-16,133

DYNAMICS OF THE MOTION OF A BODY WITH ALLOWANCE FOR THE UNSTEADY STATE OF THE FLOW ABOUT IT

S.M. Belotserkovskiy, Yu.A. Kochetkov, and V.K. Tomshin

Translation of "Dinamika dvizheniya tela pri uchete nestatsionarnosti obtekaniya," Izvestiya Akademiya Nauk SSR, Mekhanika Tverdogo Tela, May-June 1974, pp. 35-43

(NASA-TT-F-16133) DYNAMICS OF THE MOTION OF A BODY WITH ALLOWANCE FOR THE UNSTEADY STATE OF THE FLOW ABOUT IT (Kanner (Leo) Associates) 17 p HC $3.25 CSCL 01A
N75-19179
Unclas
G3/02 14448

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 FEBRUARY 1975

STANDARD TITLE PAGE

1. Report No. NASA TT F-16,133	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle DYNAMICS OF THE MOTION OF A BODY WITH ALLOWANCE FOR THE UNSTEADY STATE OF THE FLOW ABOUT IT		5. Report Date February 1975
		6. Performing Organization Code
7. Author(s) S.M. Belotserkovskiy, Yu.A. Kochetkov, and V.K. Tomshin, Moscow		8. Performing Organization Report No.
		10. Work Unit No.
9. Performing Organization Name and Address Leo Kanner Associates Redwood City, California 94063		11. Contract or Grant No. NASw-2481
		13. Type of Report and Period Covered Translation
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration, Washington, D.C. 20546		14. Sponsoring Agency Code

15. Supplementary Notes

Translation of "Dinamika dvizheniya tela pri uchete nestatsionarnosti obtekaniya," Izvestiya Akademiya Nauk SSR, Mekhanika Tverdogo Tela, May-June 1974, pp. 35-43

(A74-41490)

16. Abstract A numerical method and an approximate method based on the approximation of transient aerodynamic functions by exponential curves are proposed for the solution of the integrodifferential equations to which some dynamics problems reduce. The approximate method leads to systems of ordinary differential equations with constant coefficients. The methods are applied to solving three problems in flight dynamics. Solutions of exact equations are analyzed.

17. Key Words (Selected by Author(s))		18. Distribution Statement Unclassified-Unlimited	
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 15	22. Price

NASA-HQ

DYNAMICS OF THE MOTION OF A BODY WITH ALLOWANCE FOR THE UNSTEADY STATE OF THE FLOW ABOUT IT

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Two methods of solving integrodifferential equations are examined -- a numerical and an approximate method based on approximating transient aerodynamic functions with exponential curves. The latter approach leads to systems of ordinary differential equations with constant coefficients. These methods are used in solving three problems in dynamics. Solutions of the exact equations are analyzed. These data are compared with the results obtained for the hypothesis of steady-state conditions. /35*

Several problems in dynamics involve a detailed study of transient processes occurring when flight vehicles of different functions travel in a continuous medium. Thus far they have been solved even for linearized equations by resorting to additional hypotheses, usually steady-state conditions or harmonic conditions [1-3]. The study of the action of a turbulent medium and high-frequency oscillations of vehicle parts and its covering, and rapid deflections of control surfaces compensating for these actions and preventing auto-oscillations requires revisions of existing approaches. Aeroautoelasticity is concerned with investigating these processes, with methods of flight vehicle control, and with suppressing oscillations in vehicle parts by means of automatic control systems. As a rule, aeroautoelasticity can be based on linear concepts, when dimensionless kinematic parameters (angles of attack and slip, deflections of control surfaces, angular velocities of craft, and deformations) can be assumed to be small compared with unity and it is possible to

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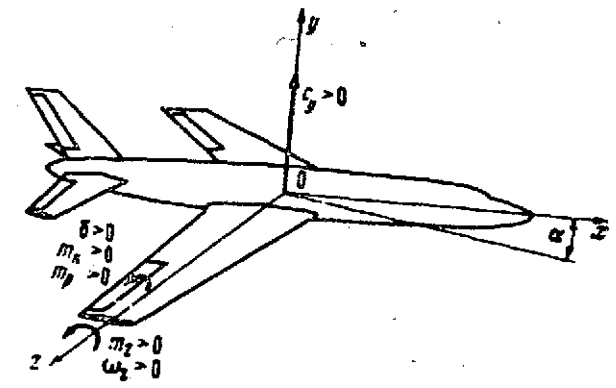

Fig. 1

adhere only to their first powers. This permits exact linear equations to be obtained describing these phenomena and relating them to nonsteady processes of flow of liquid or gas past the vehicle [4].

This article studies the linear integrodifferential equations obtained; here two methods of solving are proposed -- a linear and an approximate method based on approximating transient aerodynamic functions with exponential curves. The latter leads to linear systems of ordinary differential equations.

Three specific problems in dynamics are studied by these techniques: rotational motion of the vehicle, its translational oscillations, and the deflection of a control surface acted on by a given torque of the drive. It is assumed that during the transient process the vehicle velocity is the same. Solutions of exact equations are analyzed; in particular, they are compared with results based on the hypothesis of steady-state conditions.

1. Suppose a nonsteady process originates when t = 0. We /36
introduce the dimensionless time

$$\tau = \frac{u_0 t}{b} \qquad (1.1)$$

where u_0 is the mean translational velocity of the body, and b is the characteristic linear dimension (for wing or control surface -- their largest chord).

Let $q_i(\tau)$ stand for the dimensionless kinematic parameters. For a solid moving in the plane Oxy (Fig. 1), it follows that according to [3] we take

$$q_1=\alpha(\tau),\qquad q_2=\omega_z(\tau)=\frac{\Omega_z b}{u_0},\qquad q_3=\delta(\tau),\qquad q_4=\delta^{\cdot}(\tau)=\frac{d\delta}{d\tau} \tag{1.2}$$

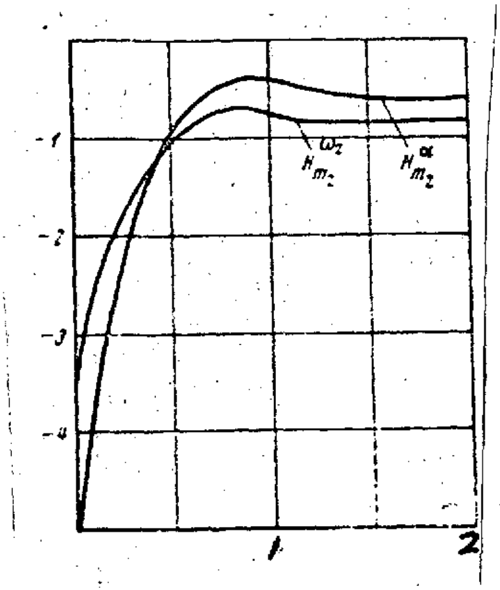

Fig. 2.

Here α is the angle of attack, Ω_z is the angular velocity of the body around the axis Oz and δ is the angle of control surface deflection.

Further, we need the overall aerodynamic characteristics of the body $c(\tau)$ (coefficients of lift c_y, longitudinal moment m_z, and hinge moment of control surface m_p). We will take area as the characteristic area S for the wing or control surface. The normalized conversion functions of these characteristics $c_{q_i}(\tau)$ when only the kinematic parameter $q_i(\tau)$ is distinct from zero is denoted with $H_c^{q_i}(\tau)$. These functions correspond to the stepwise variation of the kinematic parameter

$$q_i(\tau)=\begin{cases}0 & \text{when } \tau<0\\ 1 & \text{when } \tau>0\end{cases} \tag{1.3}$$

For any of these coefficients $c(\tau)$, the following integral representations obtain, which are exact within the frame of reference of linear representations [3]L

$$c(\tau)=\sum_i\left[c_{q_i}(\tau)+c^{q_i}q_i(-0)\right]$$
$$c_{q_i}(\tau)=H_c^{q_i}(\tau)\left[q_i(+0)-q_i(-0)\right]+\int_0^{\tau}q_i^{\cdot}(\tau-\tau_1)H_c^{q_i}(\tau_1)\,d\tau_1 \tag{1.4}$$

The quantity $c_{q_i}(\tau)$ can be expressed also in terms of the weighting function $g_c^{q_i}(\tau)$ of the aerodynamic coefficient c_i

$$c_{q_i}(\tau)=\int_{-\infty}^{\tau} g_c^{q_i}(\tau-\tau_1)\, q_i(\tau_1)\, d\tau_1=\int_{0}^{\infty} g_c^{q_i}(\tau_1)\, q_i(\tau-\tau_1)\, d\tau_1$$

The weighting function $g_c^{q_i}(\tau)$ is equal to the characteristic $c_{q_i}(\tau)$ when the kinematic parameter $q_i(\tau)$ is a unit impulse, and all the remaining kinematic parameters are equal to zero.

Eqs. (1.4) are obtained for the case when $q_i(\tau)$ varies continuous when $\tau > 0$ and has a discontinuity $q_i(+0) - q_i(-0)$ when $\tau = 0$. The steady aerodynamic derivative is denoted by c^{q_i}. If /37
we use the hypothesis of steady-state conditions, instead of Eq. (1.4) we will have [3]

$$c(\tau)=\sum_i c^{q_i} q_i(\tau) \tag{1.5}$$

Fig. 2 presents the transient functions of the coefficient of the longitudinal moment of a parallel wing having aspect ratio $\lambda = 2.5$ for Mach number $M = 0.4$ They refer to the origin of coordinates O coinciding with the leading edge of the wing. The corresponding aerodynamic derivatives are

$$m_z^{\alpha}=-0.68, \quad m_z^{\omega_z}=-0.96$$

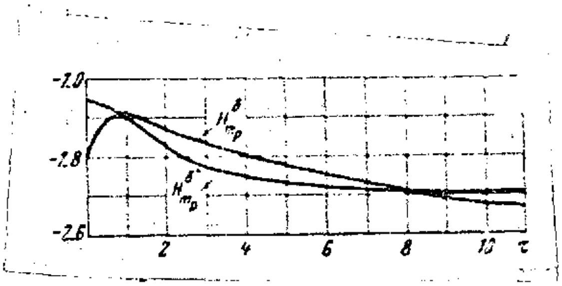

Fig. 3.

The transient functions of the hinge moment of the control surfaces of rectangular form in plan view are shown in Fig. 3 (M = 1.1) and Fig. 4 (M = 2.0). The aspect ratio of each of the control surfaces symmetrically arranged on the wing is $\lambda = 2.0$. The axis of their rotation is parallel to Oz and coincides with

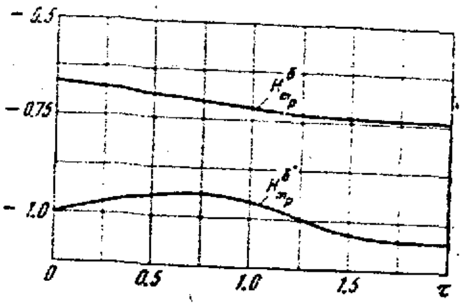

Fig. 4.

the leading edges. At supersonic velocities the planform of a thin wing only partially influences m_p. All the data presented were obtained on the assumption that the trailing edge of the wing is linear and coincides with the trailing edge of the control surfaces. The control surfaces are situated sufficiently far one from the other that there is no mutual interaction between them. Their exterior ends coincide with the lateral edges of the wing.

We have the following for the aerodynamic derivatives corresponding to the hypothesis of steady-state conditions:

$$m_p^{\delta}=-2.32 \quad m_p^{\delta'}=-2.18 \quad \text{when} \quad M=1.4$$
$$m_p^{\delta}=-0.75, \quad m_p^{\delta'}=-1.05 \quad \text{when} \quad M=2.0$$

All the aerodynamic characteristics were obtained by numerical calculations based on the methods given in the monograph [3].

2. According to [4], let us examine in more detail the simplest problems in the dynamics of a flight vehicle traveling in a continuous medium.

Suppose a body moving at constant velocity u_0 rotates about the axis Oz (Fig. 1). If I_z is the moment of inertia of the body relative to this axis, M_z is the aerodynamic moment, and ΔM_z is the moment from the spring action (for a model in a wind tunnel or a hydraulic channel), we have

$$I_z \frac{d^2\alpha}{dt^2} = M_z + \Delta M_z \qquad (2.1)$$

In this case

$$\omega_z(\tau)=\alpha^{\cdot}(\tau) \tag{2.2}$$

and the initial conditions of the problem are /38

$$\alpha(+0)=\alpha(-0)=\alpha_0, \quad \alpha^{\cdot}(+0)=\alpha^{\cdot}(-0)=0 \tag{2.3}$$

In dimensionless form, based on Eqs. (1.4), (2.1), and (2.3) the exact linearized equations for $\alpha(\tau)$ become

$$\begin{gathered}
\alpha^{\cdot\cdot}(\tau)-k_1\int\limits_0^\tau \alpha^{\cdot\cdot}(\tau-\tau_1)H_{m_z^{\dot{\alpha}}}(\tau_1)\,d\tau_1- \\
-k_1\int\limits_0^\tau \alpha^{\cdot}(\tau-\tau_1)H_{m_z^{\alpha}}(\tau_1)\,d\tau_1+k_2\alpha(\tau)-k_1 m_z^{\alpha}\alpha_0=0 \\
k_1=\frac{\rho S b^3}{2I_z}, \quad k_2=\frac{k_n b^2}{I_z u_0^2}
\end{gathered} \tag{2.4}$$

where k_n is the coefficient of spring elasticity.

The approximate form of these equations based on the hypothesis of steady-state conditions (1.4) is of the form

$$\alpha^{\cdot\cdot}(\tau)-k_1 m_z^{\dot{\alpha}}\alpha^{\cdot}(\tau)-k_1 m_z^{\alpha}\alpha(\tau)+k_2\alpha(\tau)=0 \tag{2.5}$$

Suppose a flight vehicle having mass m travels at constant velocity u_0 along the axis Ox and executes translation oscillations along the axis Oy (Fig. 1), then we have

$$m\frac{d^2y}{dt^2}=Y+\Delta Y \tag{2.6}$$

Here Y is the lift and ΔY is the force from spring action. Let us convert to the dimensionless quantity

$$\eta = \frac{y}{b}, \quad \eta^{\cdot} = \frac{d\eta}{d\tau} = -\alpha \tag{2.7}$$

We adopt as initial conditions

$$\eta(+0) = \eta(-0) = \eta_0, \quad \eta^{\cdot}(+0) = \eta^{\cdot}(-0) = 0 \tag{2.8}$$

Then the equation for the function $\eta(\tau)$ based on Eqs. (1.4), (2.6) and (2.7) will be

$$\begin{gathered} \eta^{\cdot\cdot}(\tau) + k_1' \int_0^{\tau} \eta^{\cdot\cdot}(\tau - \tau_1) H_{c_y}{}^{\alpha}(\tau_1)\, d\tau_1 + k_2' \eta(\tau) = 0 \\ k_1' = \frac{\rho S b^2}{2m}, \quad k_2' = \frac{k_n b^2}{m u_0^2} \end{gathered} \tag{2.9}$$

The approximate equation is obtained by means of (1.5), (2.6) and (2.7) and is of the form

$$\eta^{\cdot\cdot}(\tau) + k_1' c_y{}^{\alpha} \eta^{\cdot}(\tau) + k_2' \eta(\tau) = 0 \tag{2.10}$$

We obtain equations describing the motion of a control surface acted on by a given torque of the drive $M_k(t)$. Let I_0 be the moment of inertia of the control surface relative to the axis of hinges, and let M_p be the aerodynamic hinge moment acting /39 on the control surface (Fig. 1), then we have

$$I_0 \frac{d^2\delta}{dt^2} = M_p + M_k \tag{2.11}$$

We adopt as the initial conditions of control surface motion

$$\delta(+0) = \delta(-0) = 0, \quad \delta^{\cdot}(+0) = \delta^{\cdot}(-0) = 0 \tag{2.12}$$

Suppose that during control surface deflection, the velocity of the origin O of the traveling system of coordinates, attack angle, slip angle and angular velocities of the vehicle remain unchanged. If they are distinct from zero, the parts of the hinge moment corresponding to them we include in $M_k(t)$. Then based on Eqs. (1.4), (2.11) and (2.12) the exact equation for $\delta(t)$ becomes

$$
\begin{gathered}
\delta''(\tau) - k\int_0^\tau \delta''(\tau-\tau_1)\, H_{m_p}^{\delta''}(\tau_1)\,d\tau_1 - k\int_0^\tau \delta'(\tau-\tau_1)\, H_{m_p}^{\delta}(\tau_1)\,d\tau_1 - \\
- m_k(\tau) = 0 \\
k = \frac{\rho S b^3}{2I_0}, \qquad m_k = \frac{M_k b^2}{I_0 u_0^2}
\end{gathered}
\tag{2.13}
$$

By the hypothesis of steady-state conditions (1.5), instead of Eq. (2.13) we have

$$
\delta''(\tau) - k m_p^{\delta'}\delta'(\tau) - k m_p^{\delta}\delta(\tau) - m_k(\tau) = 0 \tag{2.14}
$$

3. Let us examine the system of integrodifferential equations of the following form

$$
\begin{gathered}
q_j' = f_j(q_1, \ldots, q_n; c_{q_1}, \ldots, c_{q_n}) \\
c_{q_j} = \int_0^\tau q_j'(\tau-\tau_1)\, H_c^{q_j}(\tau_1)\,d\tau_1 \qquad (j = 1, 2, \ldots, n)
\end{gathered}
\tag{3.1}
$$

The complexity of the numerical integration of this system lies in the fact that the integrand in Eq. (3.1) depends on the left-hand members of the system. Below we present a numerical method of integrating the systems, in which iteration of the process is carried out at the last step of integration.

Let $\Delta\tau$ be the step of the numerical integration of system (3.1). We set

$$c_{q_j} = g_{1j} + g_{2j}$$

$$g_{1j} = \int_0^{\Delta\tau} q_j^{*}(\tau - \tau_1) H_c^{q_j}(\tau_1)\, d\tau_1 \qquad (3.2)$$

$$g_{2j} = \int_{\Delta\tau}^{\tau} q_j^{*}(\tau - \tau_1) H_c^{q_j}(\tau_1)\, d\tau_1$$

Since the integrand in the right-hand side of the expression for g_{2j} is known, computing g_{2j} poses no difficulties. Calculation of g_{1j} is one integration step of the system (3.1) is carried out by the iteration technique. In the first approximation, we take the value of $q_j^{\cdot}$ calculated at the preceding integration step in τ; then revised by solving system (3.1). The iterative process is complete if the corrections prove to be smaller than the assigned magnitude of ε.

It turned out that the method has good convergence, since iterations are carried out over a small time interval $\Delta\tau$. Usually the number of iterations is not more than three. /40

The second approximate method of solving systems of integro-differential equations is based on an approximation of the transient functions $H_c^{q_i}(\tau)$ by linear combinations of the exponential functions

$$H_c^{q_i}(\tau) = c_{0i} + \sum_{\nu=1}^{n} c_{\nu i} e^{p_{\nu i}\tau} \qquad (3.3)$$

The transfer function of the functional

$$y_i = \int_0^{\tau} q_i^{\cdot}(\tau - \tau_1) H_c^{q_i}(\tau_1)\, d\tau_1 + q_i(-0) H_c^{q_i}(\infty) + [q_i(+0) - q_i(-0)] H(\tau) \qquad (3.4)$$

corresponding to this formula can be represented as

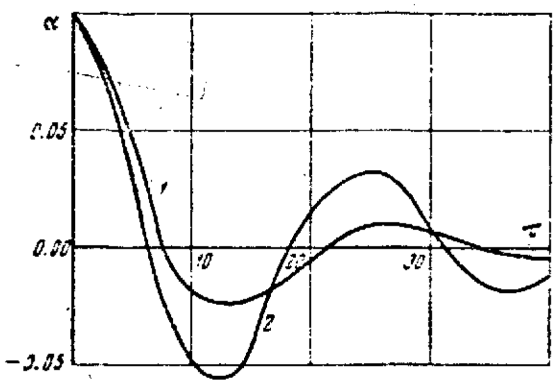

Fig. 5.

$$W_{c}^{q_i}(p)=p\left(\frac{c_{0i}}{p}+\sum_{\nu=1}^{n}\frac{c_{\nu i}}{p-p_{\nu i}}\right)=\frac{b_{ni}p^n+\ldots+b_{0i}}{a_{ni}p^n+\ldots+a_{0i}} \tag{3.5}$$

In Eqs. (3.3) and (3.5), $c_{\nu i}$ and $p_{\nu i}$ are complex numbers, while the coefficient $a_{\nu i}$ and $b_{\nu i}$ of the transfer function are real numbers. /41

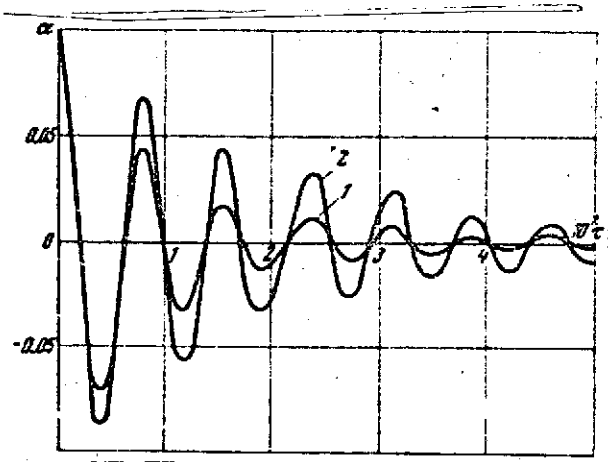

Fig. 6.

In Eq. (3.5) we separate the constant component

$$W_c^{q_i}(p)=W_{c0}^{q_i}+\frac{d_{n-1\,i}p^{n-1}+\ldots+d_{0i}}{a_{ni}p^n+\ldots+a_{0i}} \tag{3.6}$$

Then the output coordinate of the system (3.4) will be determined from the equation

$$y_i=W_{c0}^{q_i}q_i+y_{1i},\quad a_{ni}y_{1i}^{(n)}+\ldots+a_{0i}y_{1i}=d_{n-1\,i}q_i^{(n-1)}+\ldots+d_{0i}q_i \tag{3.7}$$

in which $q_i(\tau)$ is a kinematic parameter.

Thus, here instead of the integral relation (3.4) we have obtained systems of linear differential equations with constant coefficients (3.7). Calculations show that a fairly high quality of approximation is achieved even when n = 2. Therefore, each integral relation in the approximate method usually can be replaced by an ordinary second-order linear differential equation with constant coefficients.

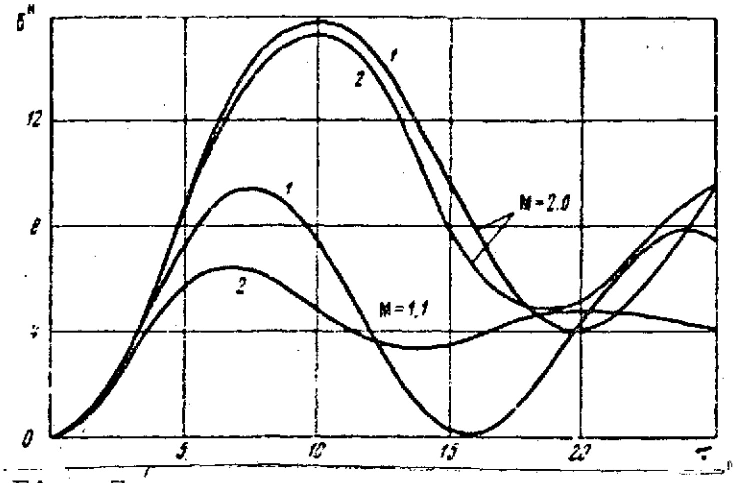

Fig. 7.

4. The numerical method of solving integro-differential equations permitted estimating the accuracy of the approximate method based on approximating transient functions. Numerous calculations showed that the difference between the solutions obtained by these methods does not exceed the error of approximation.

To investigate the effect of nonsteady-state conditions of flow on the motion of the body, all the calculations were performed in two variants: based on exact nonsteady equations and based on equations derived on the hypothesis of steady-state conditions. We present the results for the three problems in dynamics examined in Section 2.

The angular oscillations of the wing about the axis Oz extending along the leading edge for the initial angle of attack $\alpha_0 = 0.1$ were studied. Figs. 5 and 6 show how the angle of attack α varies with respect to dimensionless time $\tau = u_0 l/b$ for a parallel wing with aspect ratio $\lambda = 2.5$ for Mach number $M = 0.5$. In these cases the springs are absent ($k_2 = 0$). Fig. 5 shows the results for $k_1 = 0.1$ (a very light wing in air or a heavy wing in /42
in water), while the plots in Fig. 6 correspond to $k_1 = 0.01$ (light wing in air). The number 1 denotes the exact solutions, and the number 2 denotes solutions based on the hypothesis of steady-state conditions.

Also, angular oscillations of different wings at subsonic and supersonic flight speeds and for small values of the

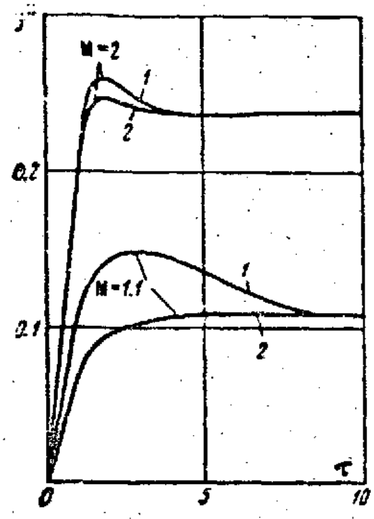

Fig. 8.

coefficient k_1 which correspond to actual mass characteristics of wings and their motion in air were analyzed. The initial conditions were taken as everywhere the same ($\alpha_0 = 0.1$); the following variants were examined.

1. Delta wing, $\lambda = 2.5$, $k_1 = 2 \cdot 10^{-3}$, $k_2 = 4 \cdot 10^{-3}$, and $M = 2.0$.

2. Delta wing, $\lambda = 2.5$, $k_1 = 2 \cdot 10^{-3}$, $k_2 = 4 \cdot 10^{-3}$, and $m = 1.2$.

3. Parallel wing, $\lambda = 5.0$, $k_1 = 10^{-3}$, $k_2 = 10^{-2}$, $M = 0.4$.

4. Parallel wing, $\lambda = 2.5$, $k_1 = 10^{-3}$, $k_2 = 10^{-2}$, $M = 0.4$.

The trend of the functions $\alpha(\tau)$ was everywhere close to that shown in Fig. 6: attenuating, almost-periodic oscillations. The periods of the oscillations based on exact theory and the hypothesis of steady-state conditions proved to be virtually identical. The values of the mean decrements of attenuation of the longitudinal oscillations are shown in Table 1.

TABLE 1

No. of variant	Exact solution	Steady-state hypothesis
1	$9.6 \cdot 10^{-4}$	$1.16 \cdot 10^{-3}$
2	$1.24 \cdot 10^{-3}$	$2.1 \cdot 10^{-3}$
3	$1.05 \cdot 10^{-3}$	$6.5 \cdot 10^{-4}$
4	$8.8 \cdot 10^{-4}$	$4.8 \cdot 10^{-4}$

Translational oscillations of the wings are characterized by the parameters $\eta_0 = 0.1$, $k_1' = 10^{-3}$, and $k_2 = 10^{-2}$, which correspond to the motion of the wing in air. The following variants were studied.

1. Parallel wing, $\lambda = 2.5$ and $M = 0.4$.

2. Parallel wing, $\lambda = \infty$ and $M = 0.4$.

3. Parallel wing, $\lambda = 2.5$ and $M = 0$ (incompressible medium).

4. Parallel wing, $\lambda = \infty$ and $M = 0$ (incompressible medium).

Here it was also established that the motion is close to attenuating periodic oscillations. Their periods based on the exact and on approximate calculations was found to be virtually the same. The values of the decrement of the translational oscillations are in Table 2.

TABLE 2

No. of variant	Exact solution	Steady-state hypothesis
1	$1.55 \cdot 10^{-3}$	$1.57 \cdot 10^{-3}$
2	$3.34 \cdot 10^{-3}$	$3.35 \cdot 10^{-3}$
3	$1.49 \cdot 10^{-3}$	$1.51 \cdot 10^{-3}$
4	$2.87 \cdot 10^{-3}$	$3.14 \cdot 10^{-3}$

In the study of forced deflection of control surfaces, the transient function of the normalized angle of deflection δ'' was analyzed, where the following was assumed:

$$m_k(\tau) = \begin{cases} 0 & \text{for } \tau < 0 \\ m_k^* & \text{for } \tau > 0, \end{cases} \qquad \delta'' = \frac{\delta}{m_k^*} \tag{4.1}$$

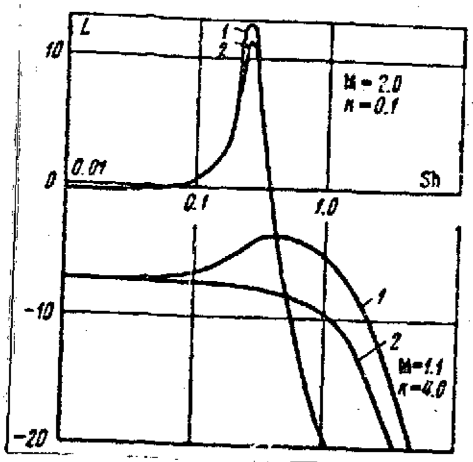

Fig. 9

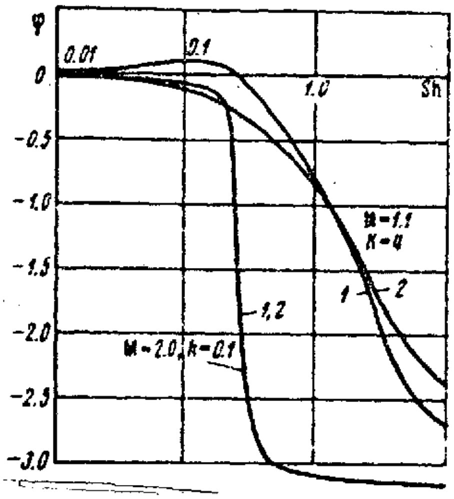

Fig. 10.

The calculations were made when M = 2.0 and M = 1.1; the coefficient k was varied from 0.1 to 4. Figs. 7 and 8 show the dependences of the normalized angle of control surface deflection δ" on the dimensionless time corresonding to a unit stepwise law of variation of $m_k(\tau)/m_k^*$ when k = 0.1 (Fig. 7) and k = 4.0 (Fig. 8). Fig.s 9 and 10 contain amplitude-frequency and phase-frequency characteristics of this system. The Strouhal number Sh = pb/u_0 was taken as the dimensionless frequency. /43

In control problems one is interested in how the increment of wing lift is delayed with respect to the action of moment $m_k(\tau)$ rotating the control surface. To answer this question, one must use the solution of $\delta''(\tau)$ obtained and the integral representation of the form (114) for $c_y\delta$ and $c_y\delta''$.

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